GLOBAL OPTIMIZATION WITH BARON

Nick Sahinidis

University of Illinois
at Urbana-Champaign

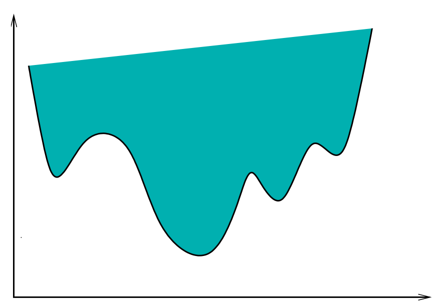
Chemical and Biomolecular Engineering

MIXED-INTEGER NONLINEAR PROGRAMMING

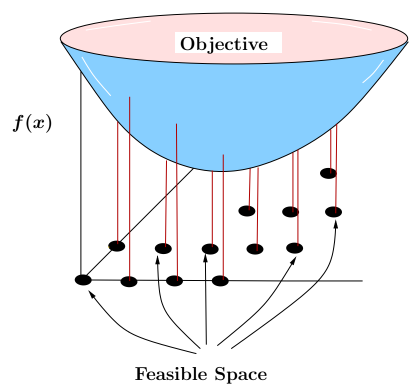
$$
\begin{array}{llll}
\text{(P)} & \min & f(x,y) & \text{Objective Function} \\
& \text{s.t.} & g(x,y) \le 0 & \text{Constraints} \\
& & x \in \mathbb{R}^n & \text{Continuous Variables} \\
& & y \in \mathbb{Z}^p & \text{Integrality Restrictions}
\end{array}
$$

Challenges:

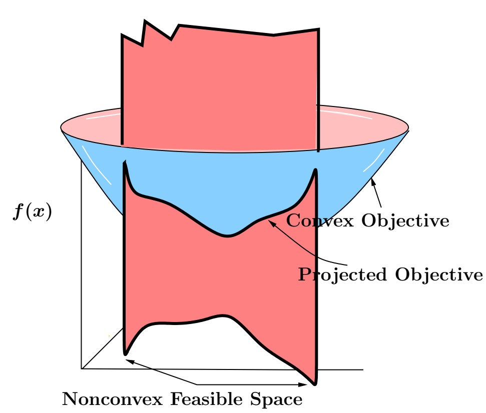
- **Multimodal Objective**



- **Integrality**



- **Nonconvex Constraints**

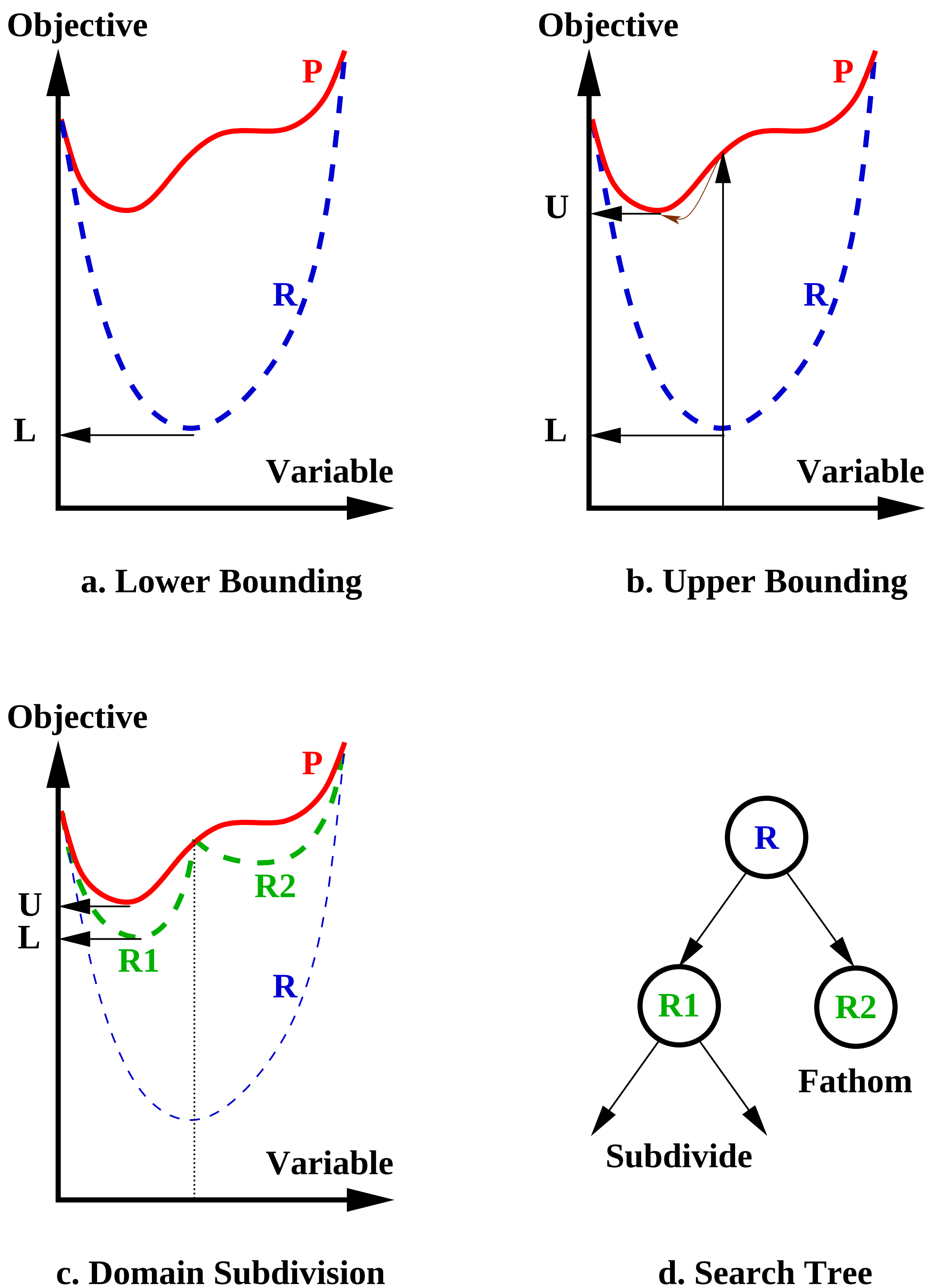


MINLP ALGORITHMS

- **Branch-and-Bound**
  - Bound problem over successively refined partitions
    - » Falk and Soland, 1969
    - » McCormick, 1976
- **Convexification**
  - Outer-approximate with increasingly tighter convex programs
  - Tuy, 1964
  - Sherali and Adams, 1994
- **Decomposition**
  - Project out some variables by solving subproblem
    - » Duran and Grossmann, 1986
    - » Visweswaran and Floudas, 1990

- **Our approach**
  - Branch-and-Reduce
    - » Ryoo and Sahinidis, 1995, 1996
    - » Shectman and Sahinidis, 1998
  - Constraint Propagation & Duality-Based Reduction
    - » Ryoo and Sahinidis, 1995, 1996
    - » Tawarmalani and Sahinidis, 2002
  - Convexification
    - » Tawarmalani and Sahinidis, 2001, 2002
- **Tawarmalani, M. and N. V. Sahinidis, *Convexification and Global Optimization in Continuous and Mixed-Integer Nonlinear Programming,* Kluwer Academic Publishers, Nov. 2002.**

BRANCH-AND-BOUND



FACTORABLE FUNCTIONS
(McCormick, 1976)

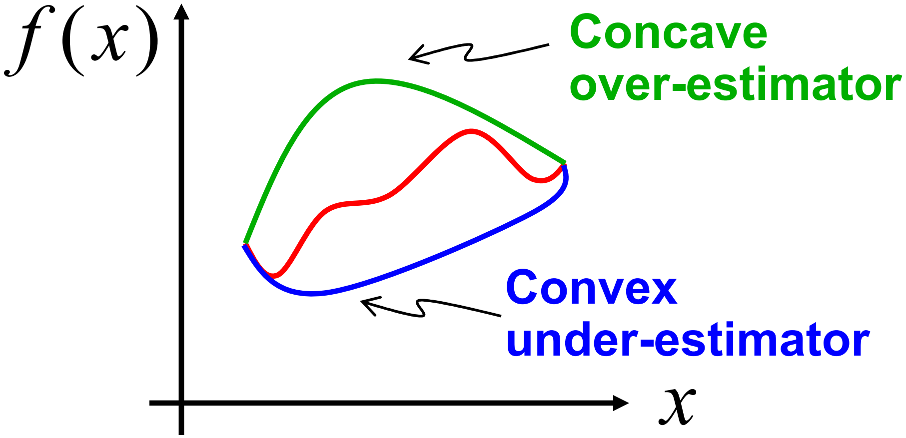
Definition: Factorable functions are recursive compositions of sums and products of functions of single variables.

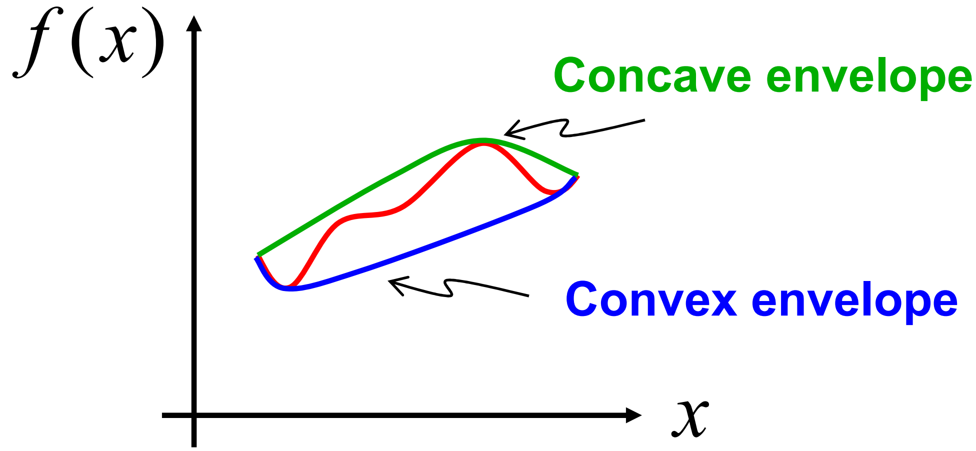
Example: $f(x, y, z, w) = \sqrt{\exp(xy + z\ln w)z^3}$

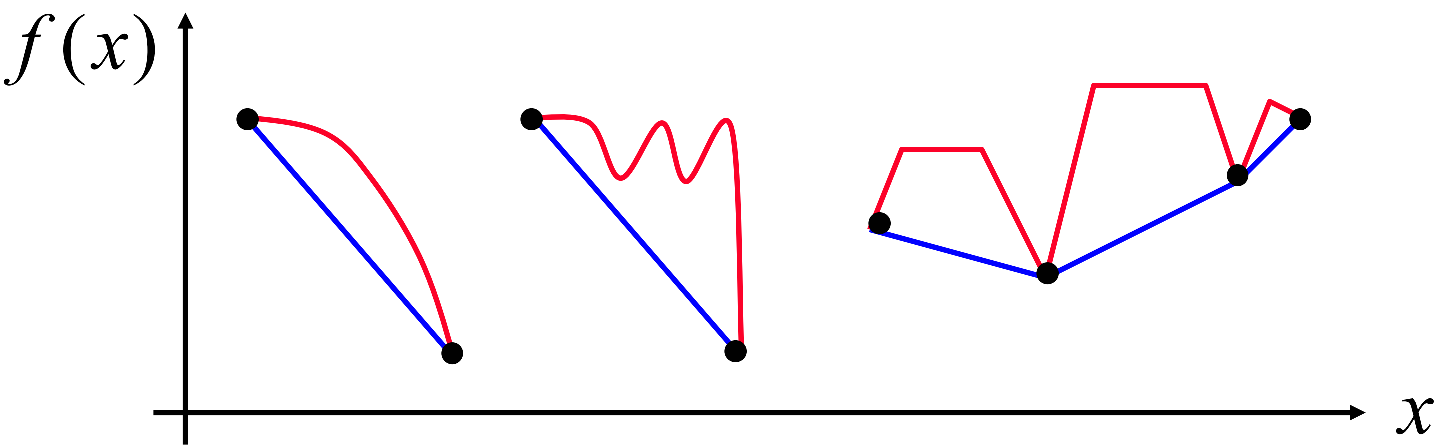
$$\overbrace{\left(\underbrace{\overbrace{\exp(\underbrace{\underbrace{xy}_{x_1} + \overbrace{z\,\underbrace{\ln w}_{x_2}}^{x_3}}_{x_4})}^{x_5}\,\overbrace{z^3}^{x_6}}_{x_7}\right)^{0.5}}^{f}$$

$$\begin{aligned}
x_1 &= xy\\
x_2 &= \ln(w)\\
x_3 &= zx_2\\
x_4 &= x_1 + x_3\\
x_5 &= \exp(x_4)\\
x_6 &= z^3\\
x_7 &= x_5x_6\\
f &= \sqrt{x_7}
\end{aligned}$$

TIGHT RELAXATIONS







Convex/concave envelopes often finitely generated

TWO-STEP CONVEX ENVELOPE CONSTRUCTION

1. **Identify generating set**
   - **Key result: A point in set X is *not* in the generating set if it is not in the generating set over a neighborhood of X that contains it**
   - **Tawarmalani and Sahinidis (2002)**

2. **Use disjunctive programming techniques to generate convex extension**
   - **Rockafellar (1970)**
   - **Balas (1974)**

ENVELOPES OF MULTILINEAR FUNCTIONS

- **Multilinear function over a box**

$$M(x_1,\ldots,x_n)=\sum_t a_t \prod_{i=1}^{p_t} x_i, \quad -\infty < L_i \le x_i \le U_i < +\infty, \quad i=1,\ldots,n$$

- **Generating set**

$$\mathrm{vert}\left(\prod_{i=1}^{n}[L_i, U_i]\right)$$

- **Polyhedral convex encloser follows trivially from polyhedral representation theorems**

PRODUCT DISAGGREGATION

Consider the function:

$$\phi(x; y_1, \ldots, y_n) = a_0 + \sum_{k=1}^{n} a_k y_k + x b_0 + x \sum_{k=1}^{n} b_k y_k$$

Let

$$H = [x^L, x^U] \times \prod_{k=1}^{n} [y_k^L, y_k^U]$$

Then

$$\operatorname{convenv}_H \phi = \quad a_0 + \sum_{k=1}^{n} a_k y_k + x b_0 + \sum_{k=1}^{n} \operatorname{convenv}_{[y_k^L, y_k^U] \times [x^L, x^U]} (b_k y_k x)$$

Disaggregated formulations are tighter

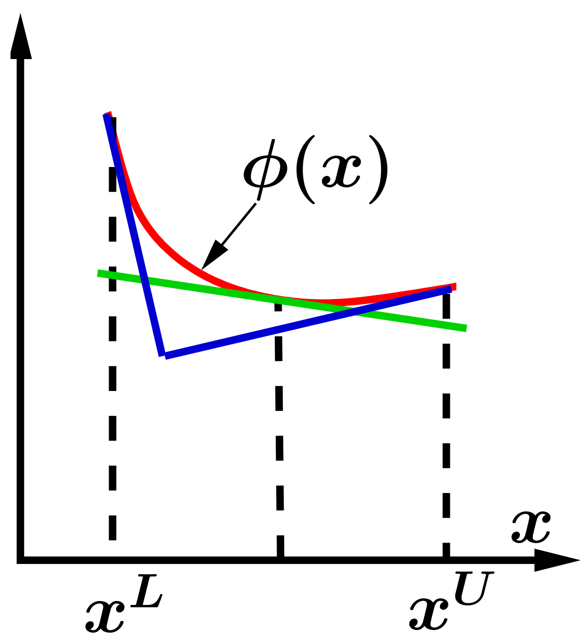
OUTER APPROXIMATION

Motivation:

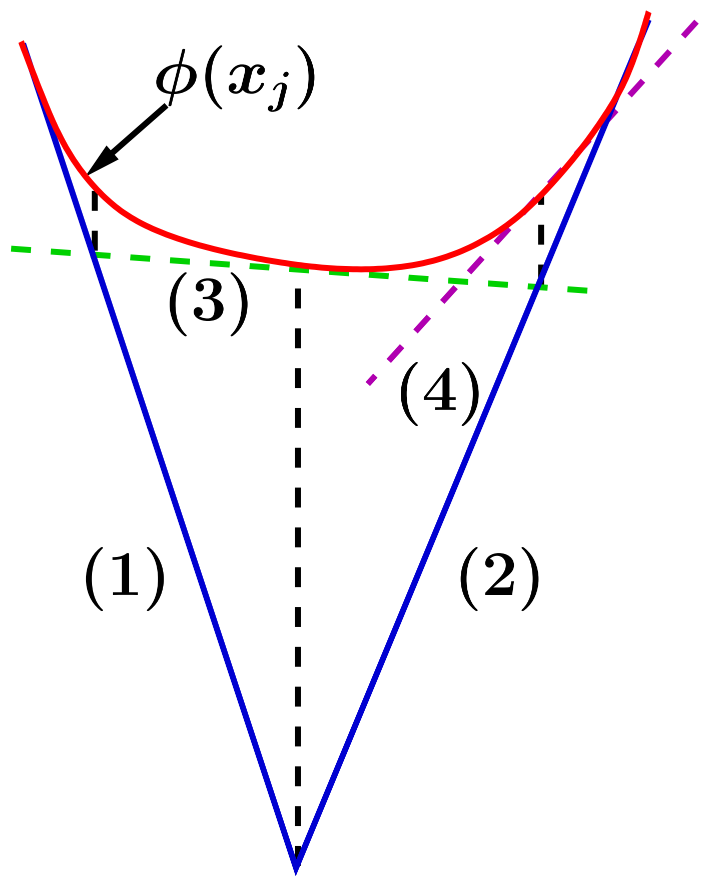
- Convex NLP solvers are not as robust as LP solvers
- Linear programs can be solved efficiently

Outer-Approximation:

Convex Functions are underestimated by tangent lines



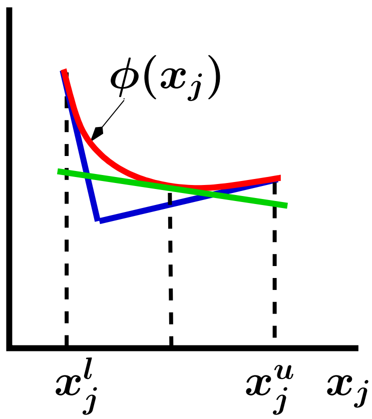
THE SANDWICH ALGORITHM



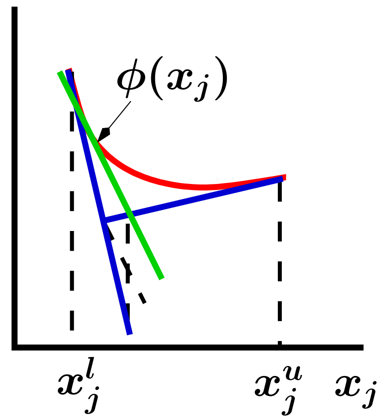
An adaptive strategy

- **Assume an initial outer-approximation**
- **Find point maximizing an error measure**
- **Construct underestimator at located point**

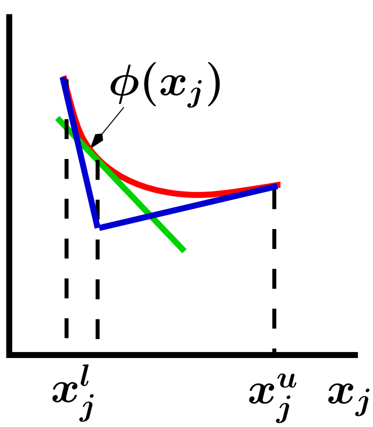
TANGENT LOCATION RULES



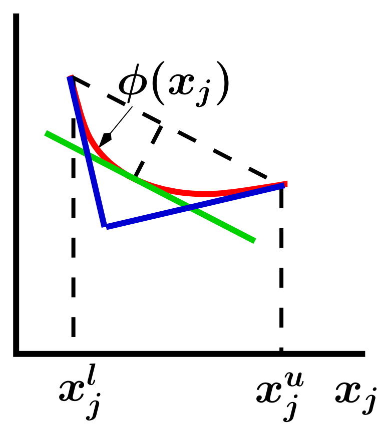
Interval bisection



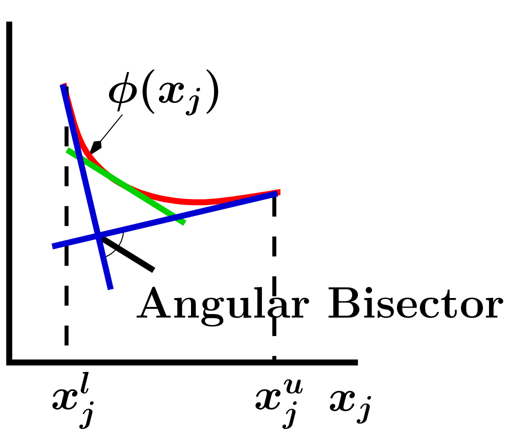
Slope Bisection



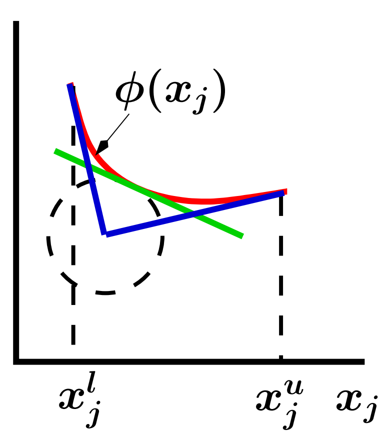
Maximum error rule



Chord rule



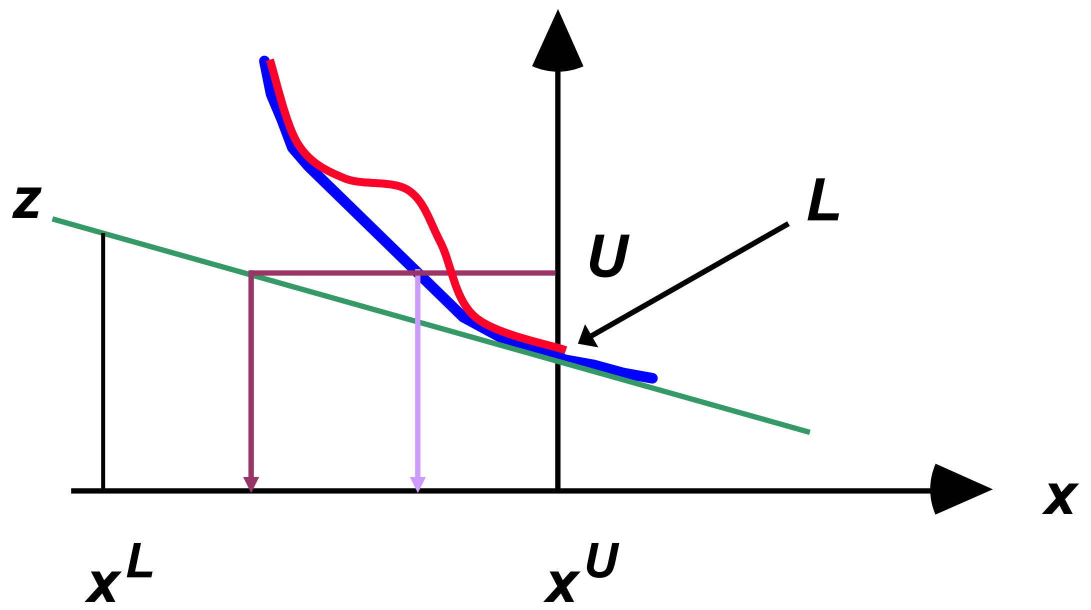
Angle bisection



Maximum projective error

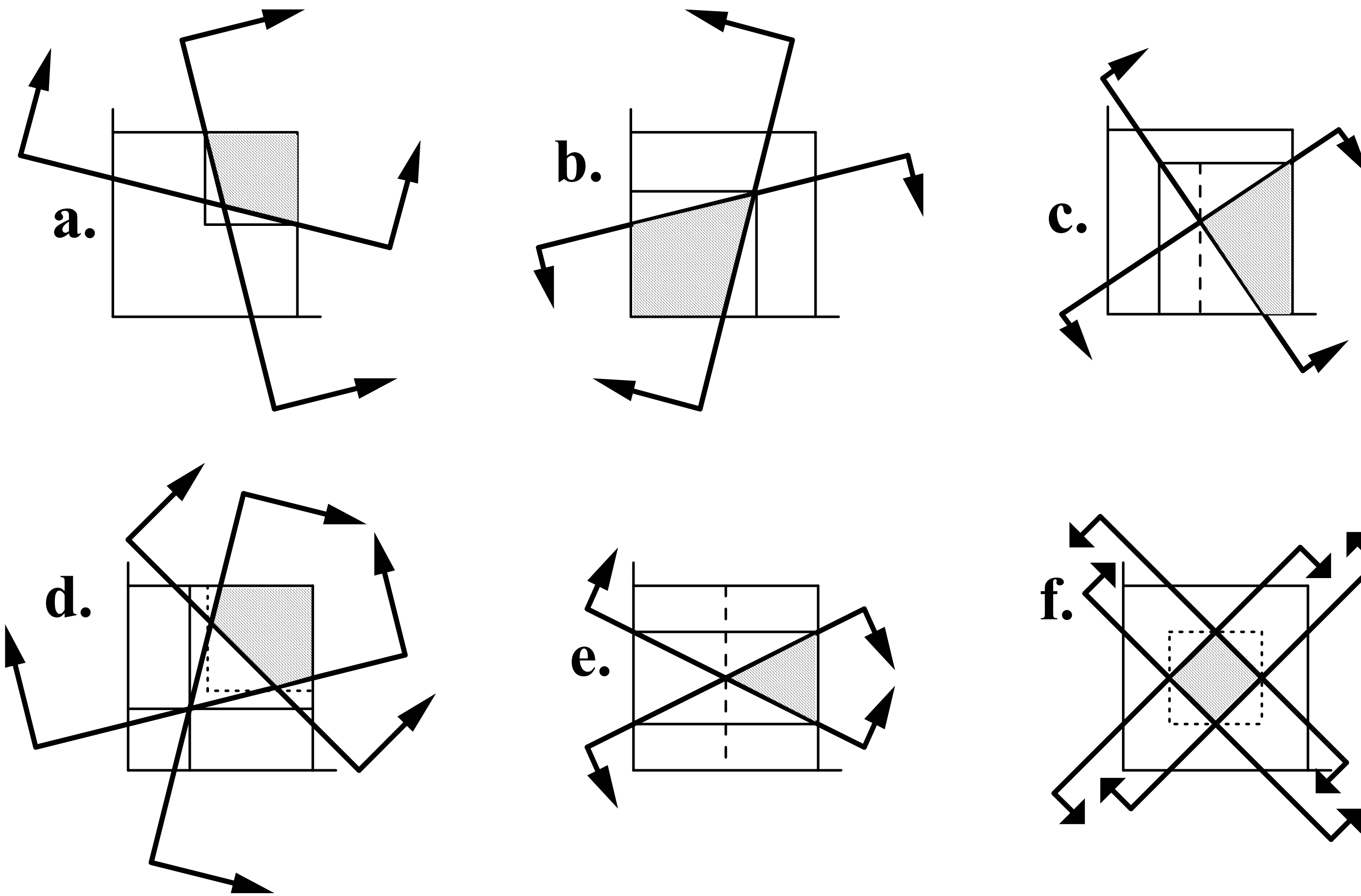
MARGINALS-BASED RANGE REDUCTION

Relaxed Value Function



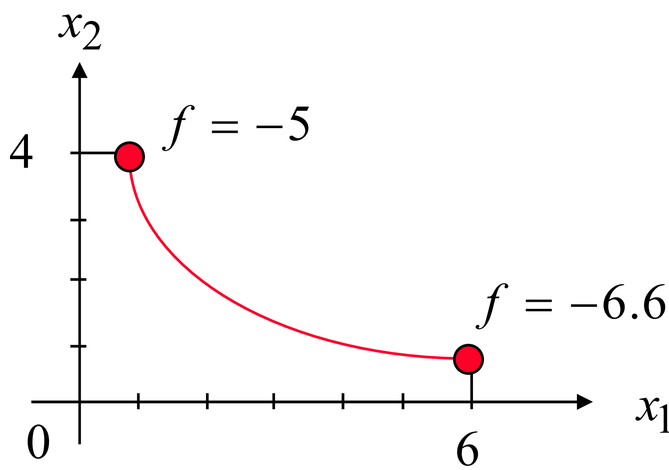
If a variable goes to its upper bound at the relaxed problem solution, this variable's lower bound can be improved

"POOR MAN'S LPs AND NLPs"



ILLUSTRATIVE EXAMPLE

$$\left.\begin{array}{ll} \min & f = -x_1 - x_2 \\ s.t. & x_1 x_2 \le 4 \\ & 0 \le x_1 \le 6 \\ & 0 \le x_2 \le 4 \end{array}\right\} \quad (P)$$



Reformulation :

$$\begin{array}{ll} \min & f = -x_1 - x_2 \\ s.t. & x_3^2 - x_1^2 - x_2^2 \le 8 \\ & x_3 = x_1 + x_2 \\ & 0 \le x_1 \le 6 \\ & 0 \le x_2 \le 4 \\ & 0 \le x_3 \le 10 \end{array}$$

Relaxation :

$$\left.\begin{array}{ll} \min & -x_1 - x_2 \\ s.t. & x_3^2 - 6x_1 - 4x_2 \le 8 \\ & x_3 = x_1 + x_2 \\ & 0 \le x_1 \le 6 \\ & 0 \le x_2 \le 4 \\ & 0 \le x_3 \le 10 \end{array}\right\} \quad (R)$$

Solution of *R* : $x_1 = 6, x_2 = 0.89, L = -6.89, \lambda_1 = 0.2$

Local Solution of *P* with MINOS : $U = -6.67$

Range reduction : $x_1^L \leftarrow x_1^U - (U - L)/\lambda_1 = 4.86$

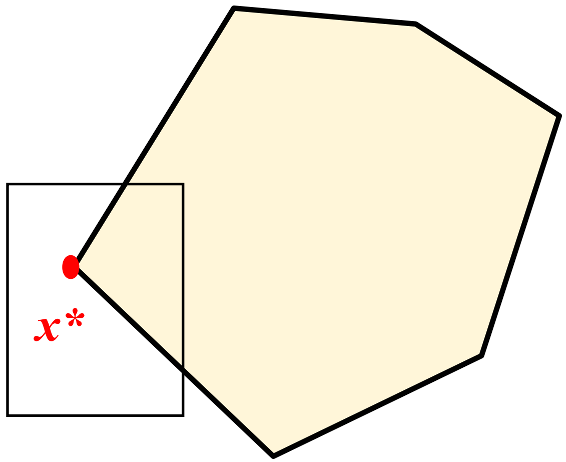
Probing (Solve R with $x_2 \le 0$) : $L = -6, \lambda_2 = 1 \Rightarrow x_2^L = 0.67$

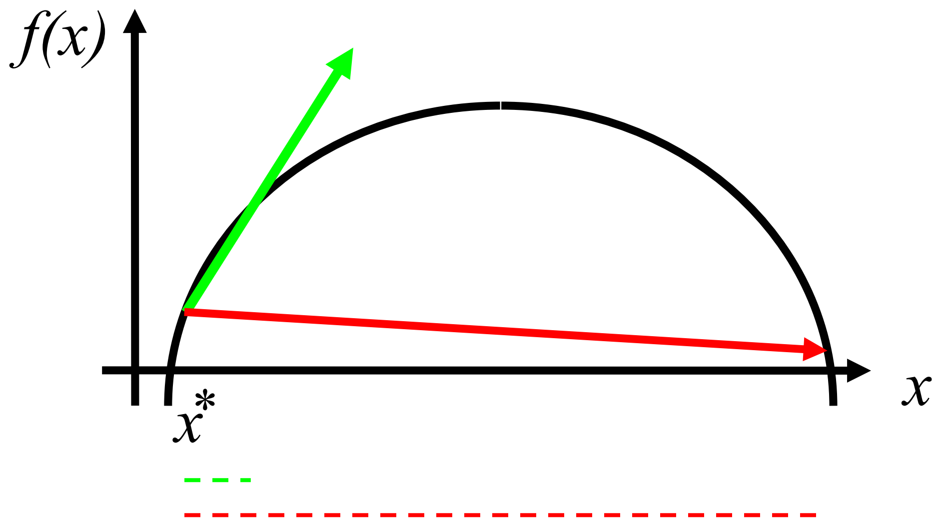
Update R with $x_1 \ge 4.86, x_2 \ge 0.67$

Solution is : $L = -6.67$

∴ Proof of globality with NO Branching!!

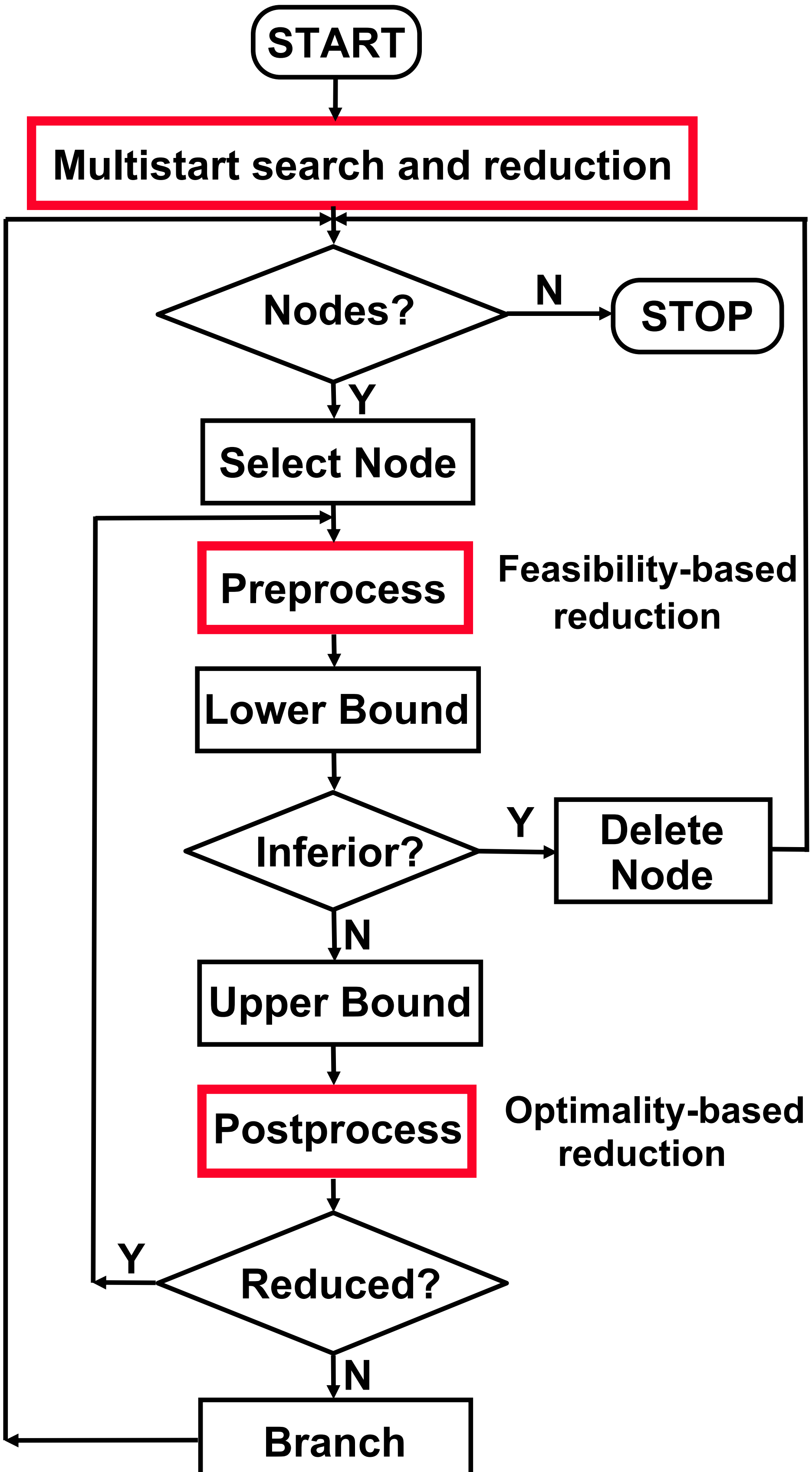
FINITE BRANCHING RULE





- **Variable selection:**
  - Typically, select variable with largest underestimating gap
  - Occasionally, select variable corresponding to largest edge
- **Locating partition:**
  - Typically, at the midpoint (exhaustiveness)
  - When possible, at the best currently known solution
    - » Makes underestimators exact at the candidate solutions
- **Finite isolation of global optimum**
- **Finite termination in many cases**

Branch-and-REDUCE



Branch-And-Reduce Optimization Navigator

Components

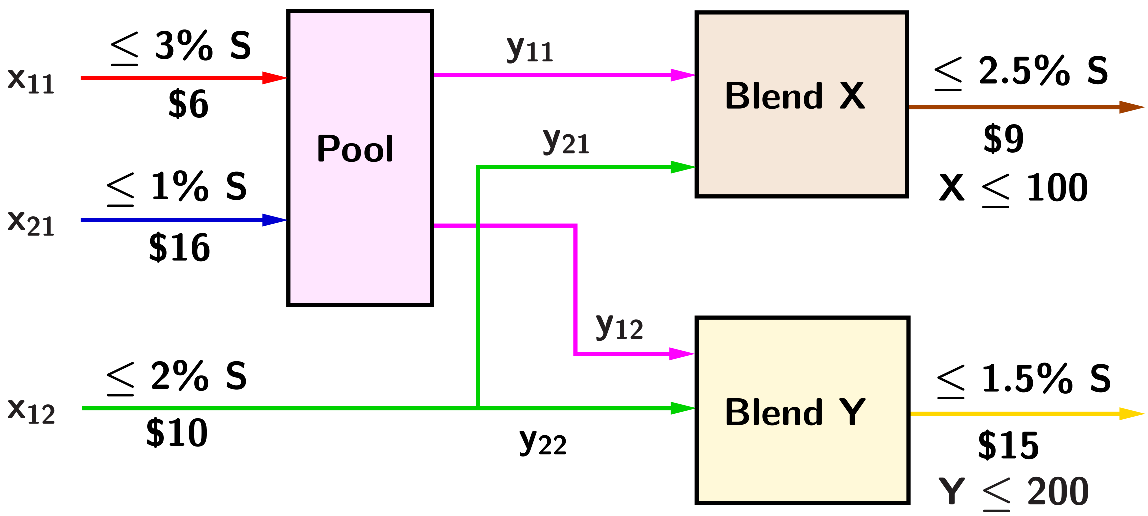
- Modeling language
- Preprocessor
- Data organizer
- I/O handler
- Range reduction
- Solver links
- Interval arithmetic
- Sparse matrix routines
- Automatic differentiator
- IEEE exception handler
- Debugging facilities

Capabilities

- Core module
  - Application-independent
  - Expandable
- Fully automated MINLP solver
- Application modules
  - Multiplicative programs
  - Indefinite QPs
  - Fixed-charge programs
  - Mixed-integer SDPs
  - …
- Solve relaxations using
  - CPLEX, MINOS, SNOPT, OSL, SDPA, …

- First on the Internet in March 1995
- On-line solver between October 1999 and May 2003
  - Solved eight problems a day
- Available under GAMS

POOLING: p FORMULATION



$$\min \quad \overbrace{6x_{11} + 16x_{21} + 10x_{12}}^{\text{cost}} - \overbrace{9(y_{11} + y_{21})}^{X\text{-revenue}} - \overbrace{15(y_{12} + y_{22})}^{Y\text{-revenue}}$$

$$\text{s.t.} \quad q = \frac{3x_{11} + x_{21}}{y_{11} + y_{12}} \qquad \text{Sulfur Mass Balance}$$

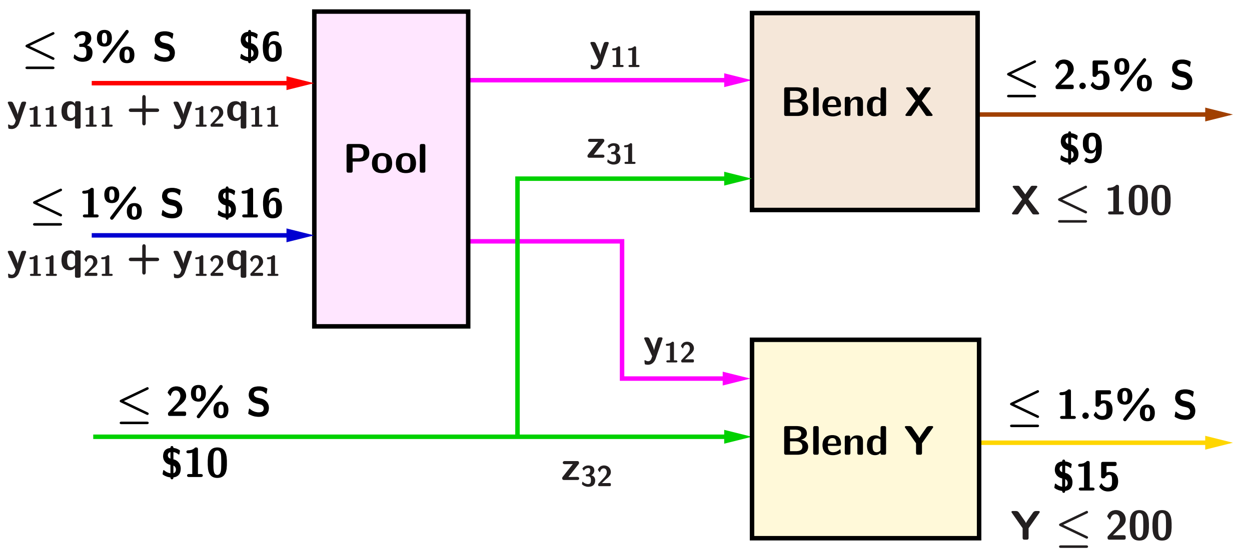
$$\begin{aligned} x_{11} + x_{21} &= y_{11} + y_{12} \\ x_{12} &= y_{21} + y_{22} \end{aligned} \qquad \text{Mass balance}$$

$$\begin{aligned} \frac{qy_{11} + 2y_{21}}{y_{11} + y_{21}} &\leq 2.5 \\ \frac{qy_{12} + 2y_{22}}{y_{12} + y_{22}} &\leq 1.5 \end{aligned} \qquad \text{Quality Requirements}$$

$$\begin{aligned} y_{11} + y_{21} &\leq 100 \\ y_{12} + y_{22} &\leq 200 \end{aligned} \qquad \text{Demands}$$

Haverly 1978

POOLING: q FORMULATION



$$\text{min} \quad \overbrace{6\,(y_{11}q_{11} + y_{12}q_{11}) + 16\,(y_{11}q_{21} + y_{12}q_{21}) + 10\,(z_{31} + z_{32})}^{\text{cost}}$$

$$- \overbrace{9(y_{11} + y_{21})}^{X\text{-revenue}} - \overbrace{15(x_{12} + x_{22})}^{Y\text{-revenue}}$$

s.t. $q_{11} + q_{21} = 1$ **Mass Balance**

$$-0.5z_{31} + 3y_{11}q_{11} + y_{11}q_{21} \le 2.5y_{11}$$
$$0.5z_{32} + 3y_{12}q_{11} + y_{12}q_{21} \le 1.5y_{12}$$

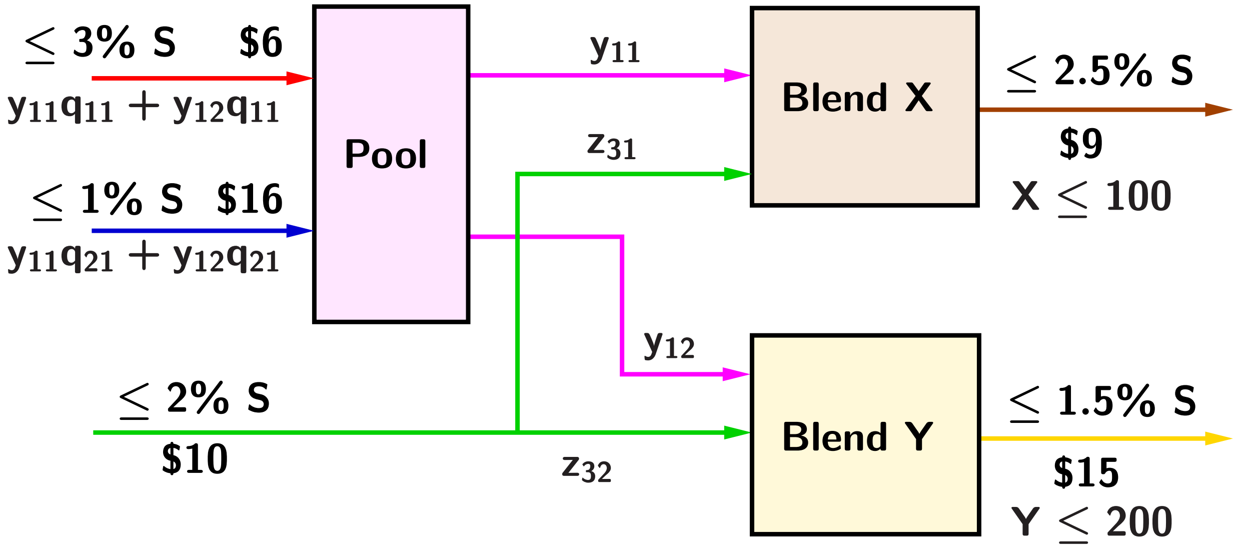
Quality Requirements

$$y_{11} + z_{31} \le 100$$
$$y_{12} + z_{32} \le 200$$

Demands

Ben-Tal et al. 1994

POOLING: pq FORMULATION



$$\min \quad \overbrace{6\,(y_{11}q_{11} + y_{12}q_{11}) + 16\,(y_{11}q_{21} + y_{12}q_{21}) + 10\,(z_{31} + z_{32})}^{\text{cost}}$$

$$- \overbrace{9(y_{11} + y_{21})}^{X\text{-revenue}} - \overbrace{15(x_{12} + x_{22})}^{Y\text{-revenue}}$$

s.t. $\quad q_{11} + q_{21} = 1 \qquad$ **Mass Balance**

$$-0.5z_{31} + 3y_{11}q_{11} + y_{11}q_{21} \le 2.5y_{11}$$
$$0.5z_{32} + 3y_{12}q_{11} + y_{12}q_{21} \le 1.5y_{12}$$

Quality Requirements

$$y_{11} + z_{31} \le 100$$
$$y_{12} + z_{32} \le 200$$

Demands

$$y_{11}q_{11} + y_{11}q_{21} = y_{11}$$
$$y_{12}q_{11} + y_{12}q_{21} = y_{12}$$

Convexification Constraints

Proof relies on Convex Extensions

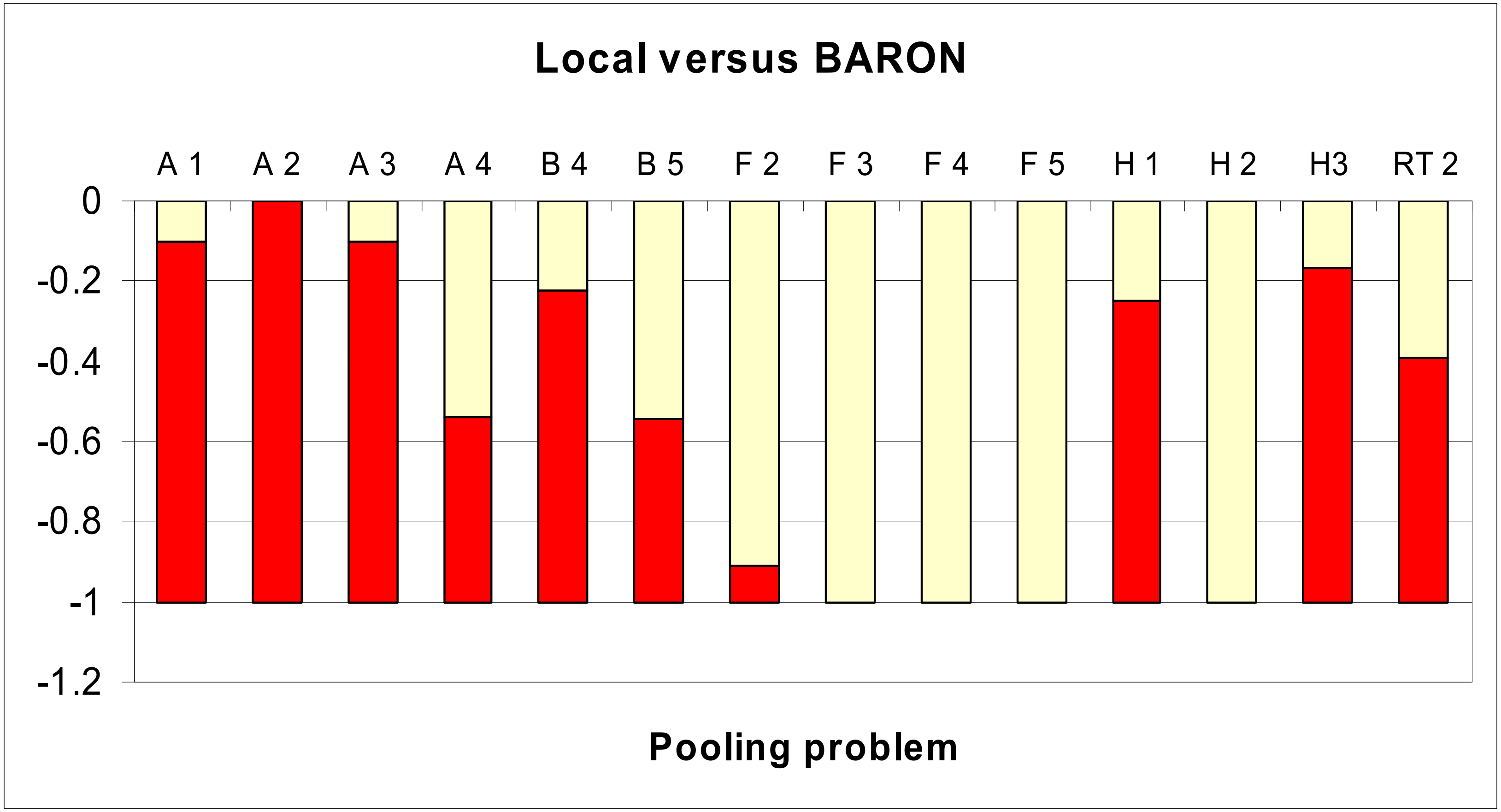
POOLING PROBLEMS

Algorithm	Foulds '92		Ben-Tal '94		GOP '96		BARON '99		BARON '01	
Computer*	CDC 4340				HP9000/730		RS6000/43P		RS6000/43P	
Linpack	> 3.5				49		59.9		59.9	
Tolerance*					**		10^{-6}		10^{-6}	
Problem	N_{tot}	T_{tot}	N_{tot}	T_{tot}	N_{tot}	T_{tot}	N_{tot}	T_{tot}	N_{tot}	T_{tot}
Haverly 1	5	0.7	3	-	12	0.22	3	0.09	1	0.09
Haverly 2			3	-	12	0.21	9	0.09	1	0.13
Haverly 3			3	-	14	0.26	5	0.13	1	0.07
Foulds 2	9	3.0					1	0.10	1	0.04
Foulds 3	1	10.5					1	2.33	1	1.70
Foulds 4	25	125.0					1	2.59	1	0.38
Foulds 5	125	163.6					1	0.86	1	0.10
Ben-Tal 4			25	-	7	0.95	3	0.11	1	0.13
Ben-Tal 5			283	-	41	5.80	1	1.12	1	1.22
Adhya 1							6174	425	15	4.00
Adhya 2							10743	1115	19	4.48
Adhya 3							79944	19314	5	3.16
Adhya 4							1980	182	1	0.97

* Blank indicates problem not reported or not solved

** 0.05% for Haverly 1, 2, 3, 0.05% for Ben-Tal 4 and 1% for Ben-Tal 5

LOCAL vs. GLOBAL SEARCH



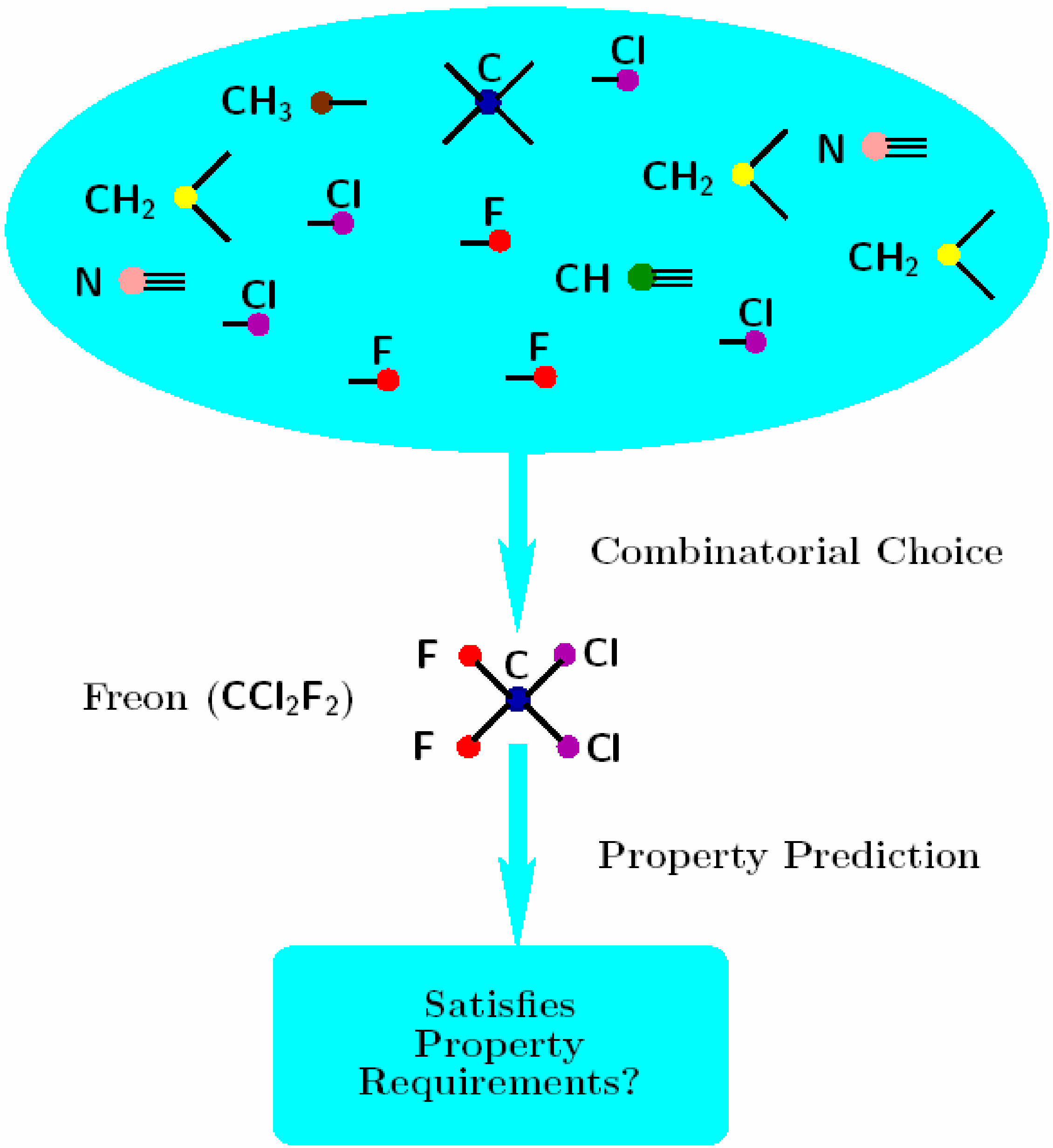
GUPTA-RAVINDRAN MINLPs

Problem		Obj.	T_{tot}	N_{tot}	N_{mem}
1		12.47	0.11	22	4
2	*	5.96	0.03	7	4
3		16.00	0.03	3	2
4		0.72	0.01	1	1
5		5.47	4.48	232	22
6		1.77	0.06	11	5
7		4.00	0.03	3	2
8		23.45	0.40	7	2
9		-43.13	0.58	37	7
10		-310.80	0.06	12	4
11		-431.00	0.12	34	8
12		-481.20	0.29	67	12

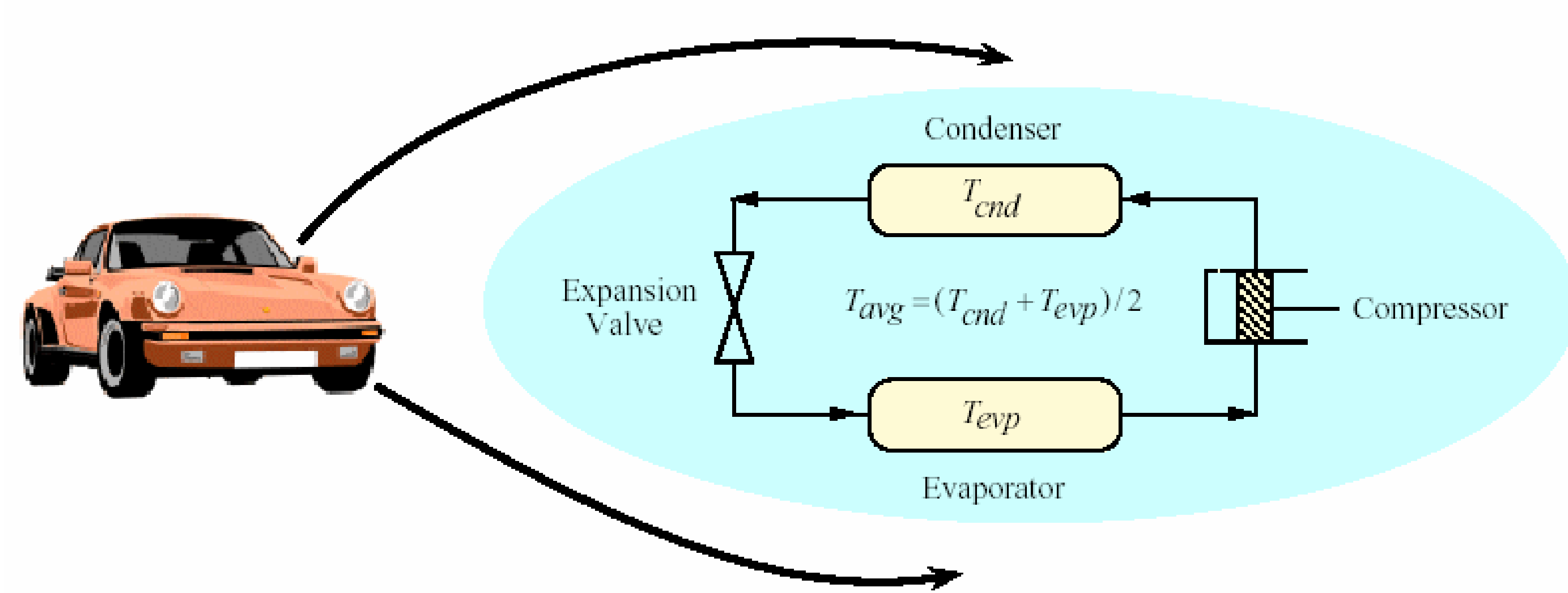
Problem		Obj.	T_{tot}	N_{tot}	N_{mem}
13		-585.20	1.13	197	28
14	*	-40358.20	0.05	7	4
15		1.00	0.05	11	3
16		0.70	0.05	23	12
17		-1100.40	42.2	3489	399
18		-778.40	8.85	993	121
19		-1098.40	133	6814	833
20	*	230.92	6.58	143	18
21	*	-5.68	0.21	54	5
22		6.06	2.36	171	39
23		-1125.20	1152	39918	4678
24		-1033.20	4404	124282	15652

* Indicates that a better solution was found than reported in Gupta and Ravindran, Man. Sci., 1985.

MOLECULAR DESIGN



AUTOMOTIVE REFRIGERANT DESIGN
(Joback and Stephanopoulos, 1990)



- Higher enthalpy of vaporization (ΔH_{ve}) reduces the amount of refrigerant
- Lower liquid heat capacity (C_{pla}) reduces amount of vapor generated in expansion valve
- Maximize ΔH_{ve}/ C_{pla}, subject to: $\Delta H_{ve} \geq 18.4$, $C_{pla} \leq 32.2$

FUNCTIONAL GROUPS CONSIDERED

Acyclic Groups	Cyclic Groups	Halogen Groups	Oxygen Groups	Nitrogen Groups	Sulfur Groups
$-CH_3$	$^{r}-CH_2-^{r}$	$-F$	$-OH$	$-NH_2$	$-SH$
$-CH_2-$	$^{r}_{r}>CH-^{r}$	$-Cl$	$-O-$	$>NH$	$-S-$
$>CH-$	$_{r}>CH-^{r}$	$-Br$	$^{r}-O-^{r}$	$^{r}_{r}>NH$	$^{r}-S-^{r}$
$>C<$	$^{r}_{r}>C<^{r}_{r}$	$-I$	$>CO$	$>N-$	
$=CH_2$	$_{r}>C<^{r}_{r}$		$^{r}_{r}>CO$	$=N-$	
$=CH-$	$>C<^{r}_{r}$		$-CHO$	$^{r}=N-^{r}$	
$=C<$	$^{r}=CH-^{r}$		$-COOH$	$-CN$	
$=C=$	$^{r}=C<^{r}_{r}$		$-COO-$	$-NO_2$	
$\equiv CH$	$^{r}=C<_{r}$		$=O$		
$\equiv C-$	$=C<^{r}_{r}$				

Number of Groups $= 44$

Maximum Selection Size $= 15$

Candidates $= 39,895,566,894,524$

PROPERTY PREDICTION

$$T_b = 198.2 + \sum_{i=1}^{N} n_i T_{bi}$$

$$T_c = \frac{T_b}{0.584 + 0.965 \sum_{i=1}^{N} n_i T_{ci} - \left(\sum_{i=1}^{N} n_i T_{ci}\right)^2}$$

$$P_c = \frac{1}{\left(0.113 + 0.0032 \sum_{i=1}^{N} n_i a_i - \sum_{i=1}^{N} n_i P_{ci}\right)^2}$$

$$C_{p0a} = \sum_{i=1}^{N} n_i C_{p0ai} - 37.93 + \left(\sum_{i=1}^{N} n_i C_{\mathrm{p0bi}} + 0.21\right) T_{\mathrm{avg}} + \left(\sum_{i=1}^{N} n_i C_{p0ci} - 3.91 \times 10^{-4}\right) T_{\mathrm{avg}}^2 + \left(\sum_{i=1}^{N} n_i C_{\mathrm{p0di}} + 2.06 \times 10^{-7}\right) T_{\mathrm{avg}}^3$$

$$T_{\mathrm{br}} = \frac{T_{\mathrm{b}}}{T_c}$$

$$T_{\mathrm{avgr}} = \frac{T_{\mathrm{avg}}}{T_c}$$

$$T_{\mathrm{cndr}} = \frac{T_{\mathrm{cnd}}}{T_c}$$

$$T_{\mathrm{evpr}} = \frac{T_{\mathrm{evp}}}{T_c}$$

$$\alpha = -5.97214 - \ln\left(\frac{P_c}{1.013}\right) + \frac{6.09648}{T_{\mathrm{br}}} + 1.28862 \ln(T_{\mathrm{br}}) - 0.169347 T_{\mathrm{br}}^6$$

$$\beta = 15.2518 - \frac{15.6875}{T_{\mathrm{br}}} - 13.4721 \ln(T_{\mathrm{br}}) + 0.43577 T_{\mathrm{br}}^6$$

$$\omega = \frac{\alpha}{\beta}$$

$$C_{\mathrm{pla}} = \frac{1}{4.1868}\left\{C_{\mathrm{p0a}} + 8.314\left[1.45 + \frac{0.45}{1 - T_{\mathrm{avgr}}} + 0.25\omega \left(17.11 + 25.2\frac{(1 - T_{\mathrm{avgr}})^{1/3}}{T_{\mathrm{avgr}}} + \frac{1.742}{1 - T_{avgr}}\right)\right]\right\}$$

$$\Delta H_{\mathrm{vb}} = 15.3 + \sum_{i=1}^{N} n_i \Delta H_{\mathrm{vbi}}$$

$$\Delta H_{\mathrm{ve}} = \Delta H_{\mathrm{vb}} \left(\frac{1 - T_{\mathrm{evp}}/T_c}{1 - T_b/T_c}\right)^{0.38}$$

$$h = \frac{T_{br} \ln(P_c/1.013)}{1 - T_{br}}$$

$$G = 0.4835 + 0.4605h$$

$$k = \frac{h/G - (1 + T_{br})}{(3 + T_{br})(1 - T_{br})^2}$$

$$\ln P_{\mathrm{vpcr}} = \frac{-G}{T_{\mathrm{cndr}}}\left[1 - T_{\mathrm{cndr}}^2 + k(3 + T_{\mathrm{cndr}})(1 - T_{\mathrm{cndr}})^3\right]$$

$$\ln P_{\mathrm{vper}} = \frac{-G}{T_{\mathrm{evpr}}}\left[1 - T_{\mathrm{evpr}}^2 + k(3 + T_{\mathrm{evpr}})(1 - T_{\mathrm{evpr}})^3\right]$$

n_i integer

MOLECULAR STRUCTURES

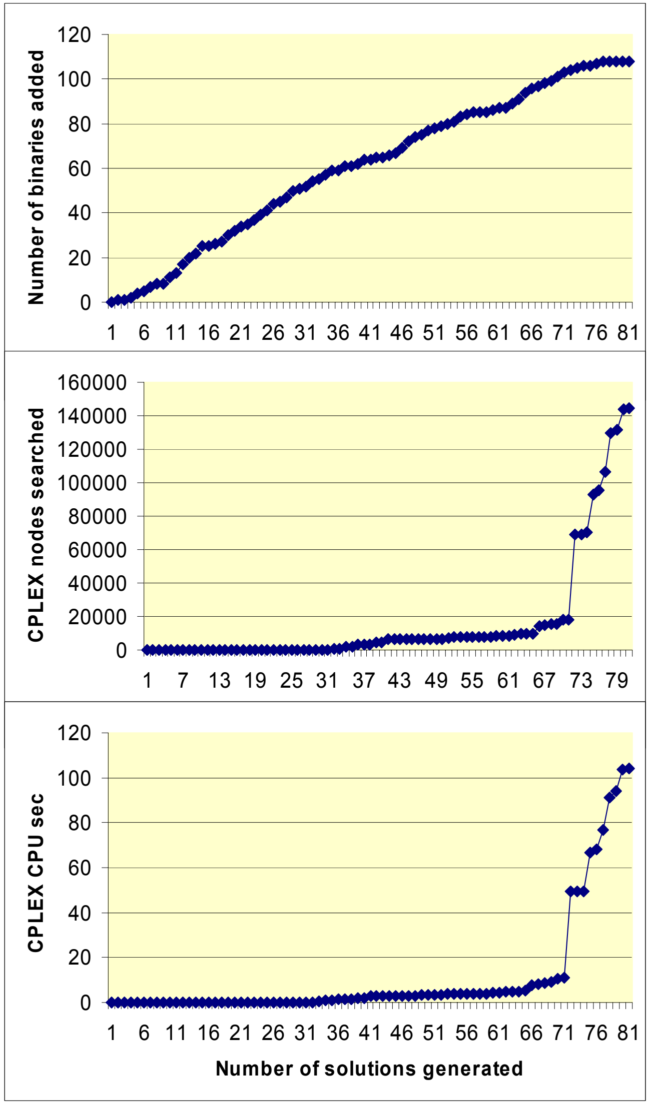
	Molecular Structure	$\frac{\Delta H_{ve}}{C_{pla}}$
FNO	$F - N = O$	1.2880
FSH	$F - SH$	1.1697
CH_3Cl	$CH_3 - Cl$	1.1219
$ClFO$	$(Cl-)(-O-)(-F)$	0.9822
C_2HClO_2	$O = C < (-CH = O)(-Cl)$	1.1207
C_3H_40	$CH_3 - CH = C = O$	0.9619
C_3H_4	$CH_3 - C \equiv CH$	0.9278
C_2F_2	$F - C \equiv C - F$	0.9229
CH_2ClF	$F - CH2 - Cl$	0.9202
C_2HO_2F	$F - O - CH = C = O$	0.8705
C_3H_4	$CH_2 = C = CH_2$	0.8656
C_2H_6	$CH_3 - CH_3$	0.8632
C_3H_3FO	$(F-)(CH3-) > C = C = O$	0.8531
NHF_2	$F - NH - F$	0.8468
C_2HOF	$CH \equiv C - O - F$	0.8263

	Molecular Structure	$\frac{\Delta H_{ve}}{C_{pla}}$
C_3H_3F	$CH \equiv C - CH_2 - F$	0.7802
CHF_2Cl	$(F-)(F-) > CH - Cl$	0.7770
C_2H_3OF	$CH_2 = CH - O - F$	0.7685
NF_2Cl	$(F-)(F-) > N - Cl$	0.7658
C_2H_6NF	$(CH3-)(CH3-) > N - F$	0.6817
N_2HF_3	$(F-)(F-) > N - NH - F$	0.6711
$C_2H_2OF_2$	$CH2 = C < (-O - F)(-F)$	0.6705
$C_3H_2F_2$	$(F-)(F-) > CH - C \equiv CH$	0.6686
C_2HNF_2	$CH \equiv C - N < (-F)(-F)$	0.6587
$C_3H_4F_2$	$(F-)(F - CH_2-) > C = CH_2$	0.6377
$C_3H_4F_2$	$(F-)(F-) > CH - CH = CH_2$	0.6263
$C_2H_3NF_2$	$CH2 = CH - N < (-F)(-F)$	0.6176
CH_3NOF_2	$(F-)(CH_3-) > N - O - F$	0.6139
$C_3H_3F_3$	(r> CH− ')$_3$(−F)$_3$	0.5977

For CCl_2F_2, $\Delta H_{ve}/C_{pla} \approx 0.57$

In 30 CPU minutes

FINDING THE *K*-BEST OR ALL FEASIBLE SOLUTIONS



Typically found through repetitive applications of branch-and-bound and generation of "integer cuts"

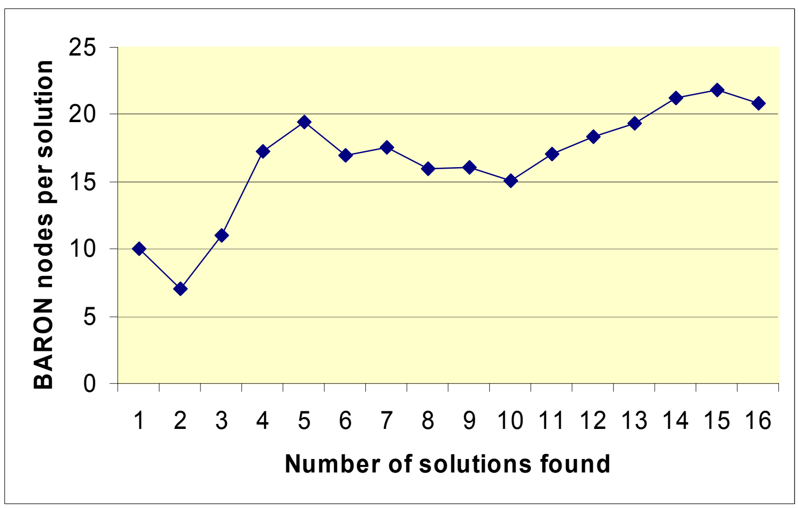
$$
\begin{aligned}
\min \quad & \sum_{i=1}^{4} 10^{4-i} x_i \\
\text{s.t.} \quad & 2 \le x_i \le 4, \qquad i = 1,\ldots,4 \\
& x \quad \text{integer}
\end{aligned}
$$

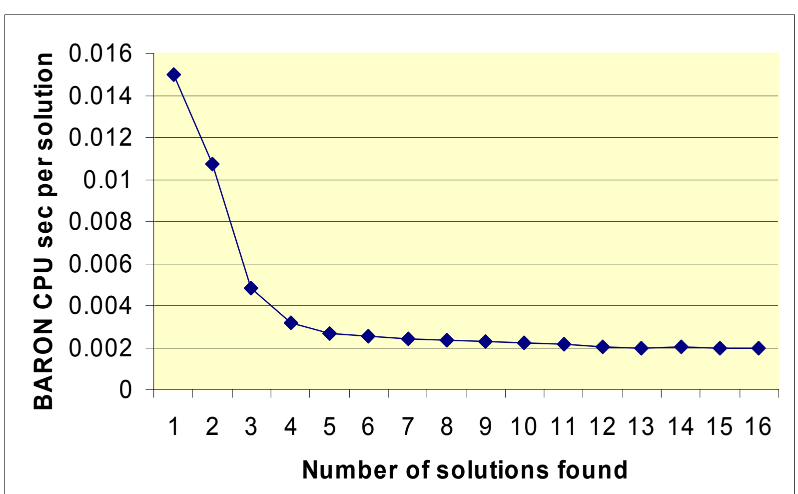
BARON finds all solutions:

- **No integer cuts**
- **Fathom nodes that are infeasible or points**
- **Single search tree**
- **511 nodes; 0.56 seconds**
- **Applicable to discrete and continuous spaces**

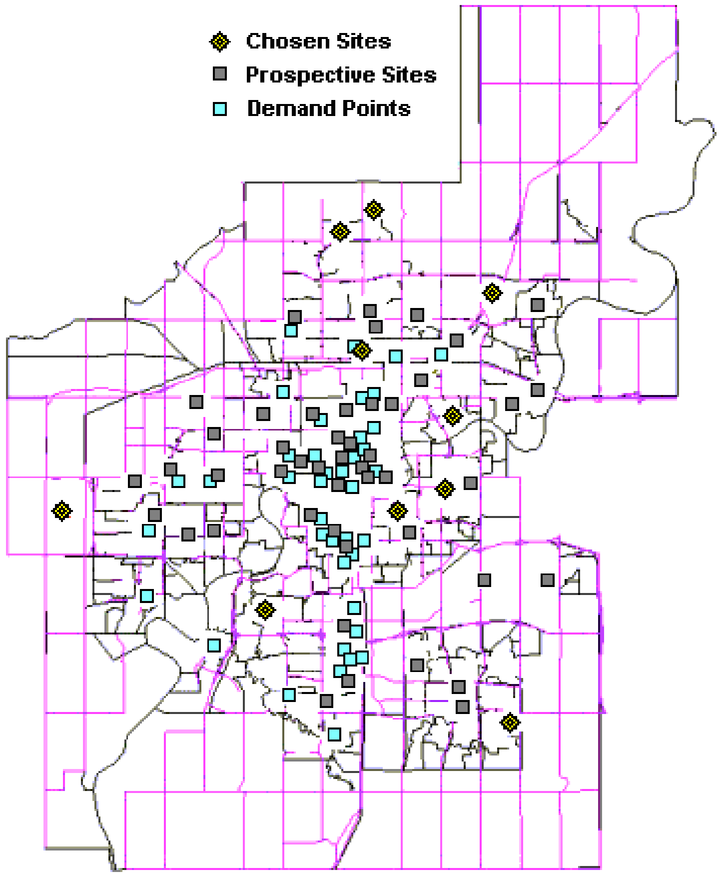
FINDING ALL or the K-BEST SOLUTIONS for CONTINUOUS PROBLEMS

- Boon problem: 8 solutions (3.1 sec)
- Robot problem: 16 solutions (0.03 sec)





Discrete Location Problems

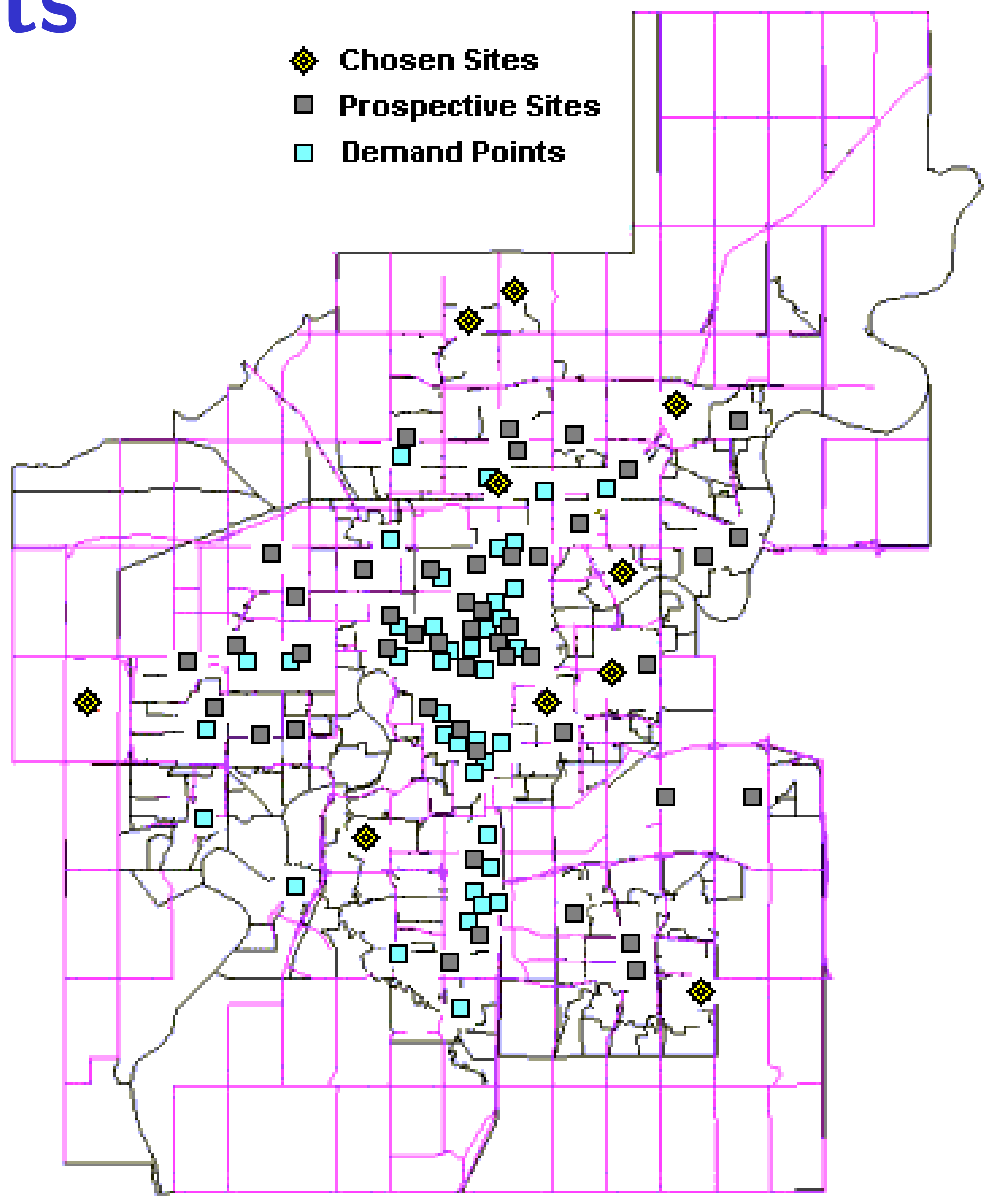


Restaurants in Edmonton

$$\textbf{p-choice Utility} = \sum_{\text{location: } j} w_j \sum_{\text{customer: } i} \overbrace{\frac{U_{ij} x_j}{\sum_k U_{ik} x_k}}^{\text{Utility Ratio}} d_i$$

U_{ij}	**Utility of location j for customer i**
x_j	**decision variable for locating facility j**
d_i	**demand by customer i**
w_j	**preferential weight of site j**

Located Restaurants



- IBM SP/2 Single Processor
- Time: 5.5 hours
- Relaxations take 95% time
- Iterations: 79
- Nodes in Memory: 9

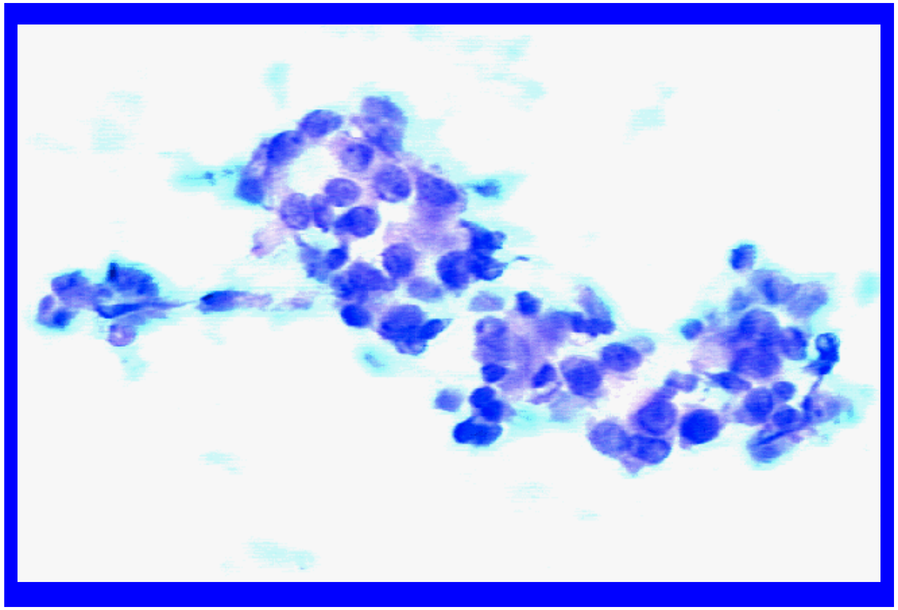
BREAST CANCER DIAGNOSIS

- 200,000 cases diagnosed in the U.S. a year
- 40,000 deaths a year

- Most breast cancers are first diagnosed by the patient as a lump in the breast
- Majority of breast lumps are benign

- Available diagnosis methods:
  - Mammography (68% to 79% correct)
  - Surgical biopsy (100% correct but invasive and costly)
  - Fine needle aspirate (FNA)
    - With visual inspection: 65% to 98% correct
    - Automated diagnosis: 95% correct
      - Linear programming techniques
      - Wolberg and Mangasarian, 1990

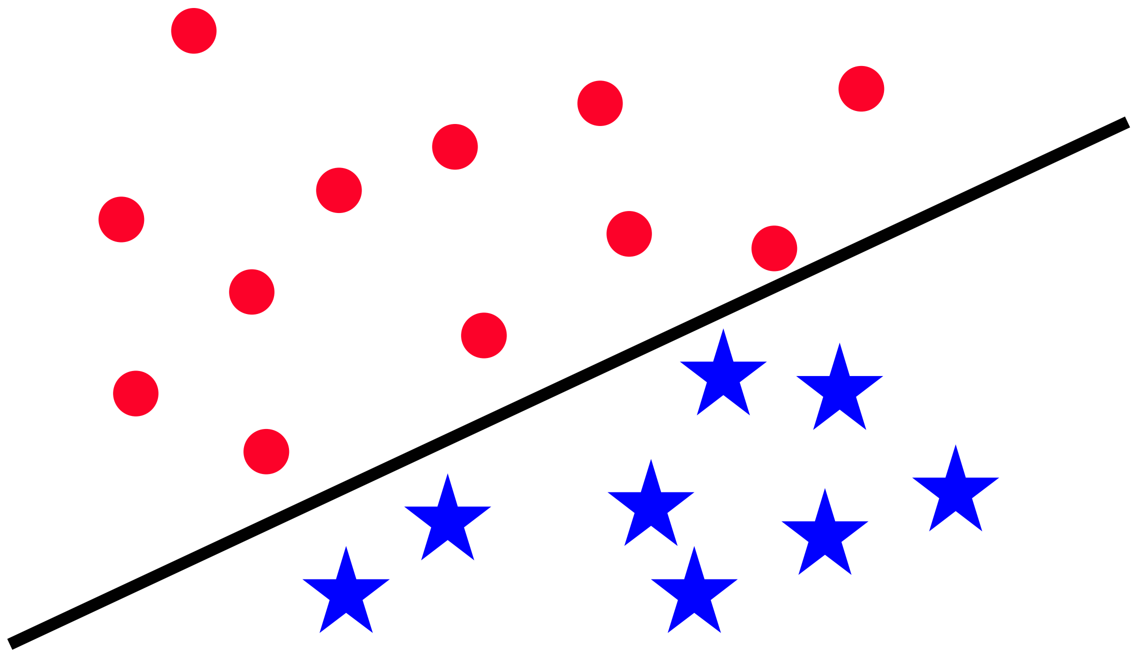
WISCONSIN DIAGNOSTIC BREAST CANCER (WDBC) DATABASE



From Wolberg, Street, & Mangasarian, 1993

- 353 FNAs (Group 1)
  - 2 Classes:
    - » 188 Benign
    - » 165 Malignant
- 9 Cytological Characteristics:
  - Clump Thickness
  - Uniformity of Cell Size
  - Uniformity of Cell Shape
  - Marginal Adhesion
  - Single Epithelial Cell Size
  - Bare Nuclei
  - Bland Chromatin
  - Normal Nucleoli
  - Mitoses
- 300 FNAs (Groups 2-8)
  - Used for testing

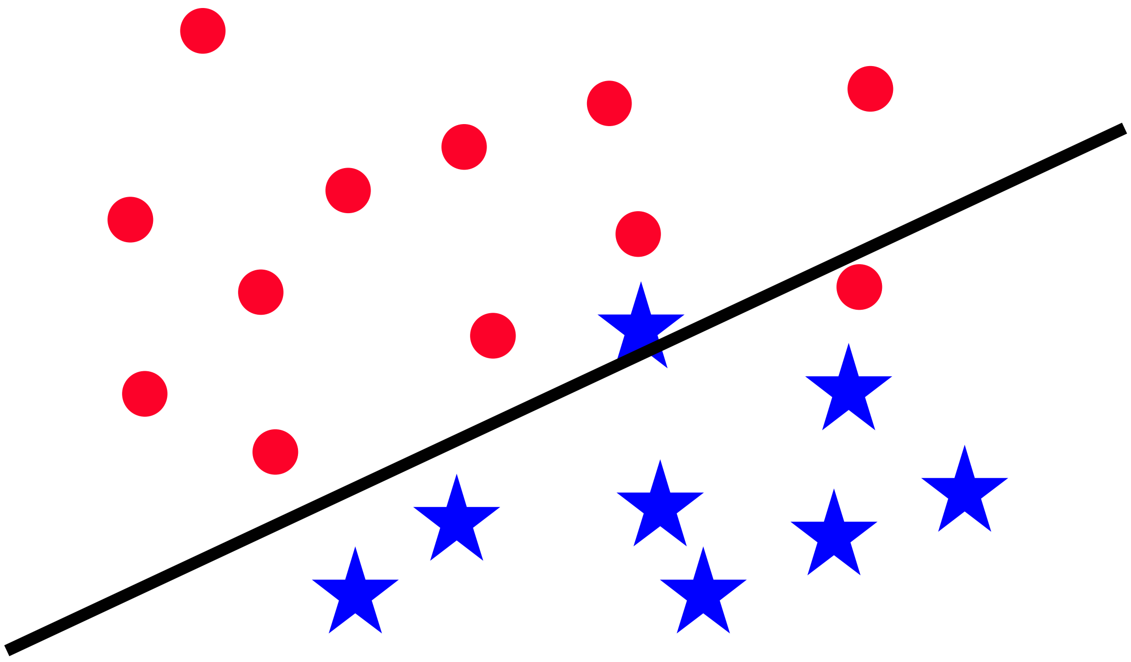
TWO SET SEPARATION IN R^n: Linearly Separable Sets



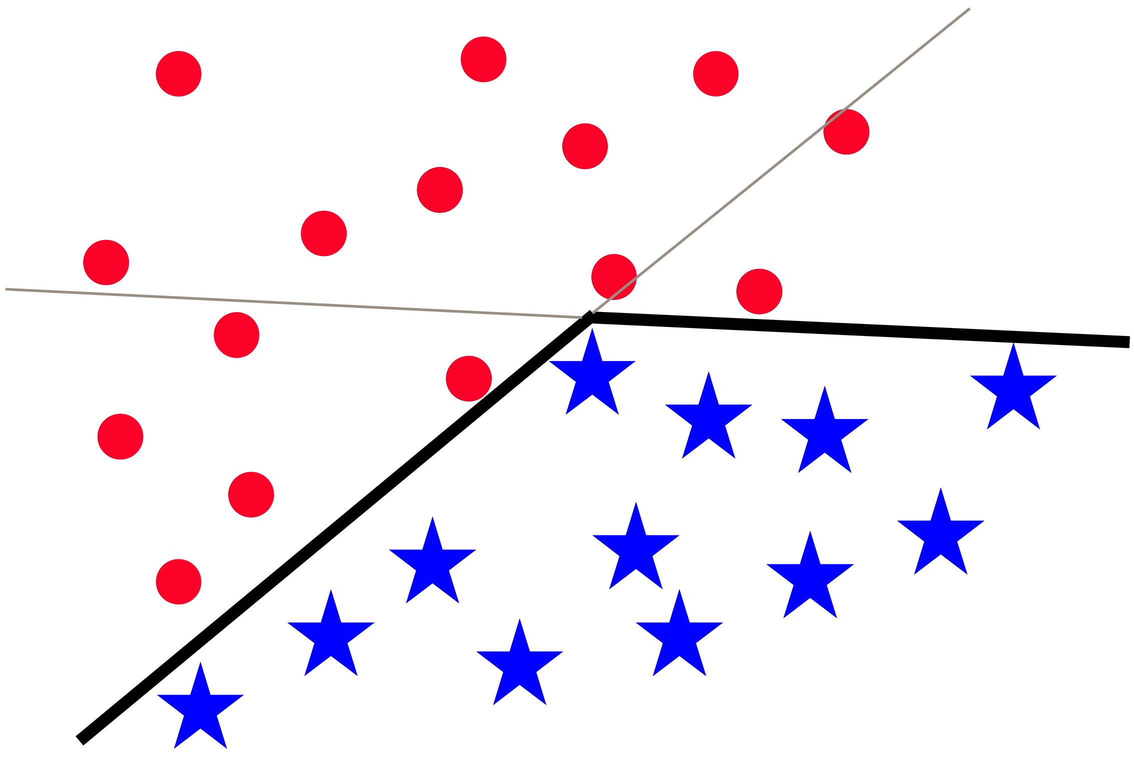
Amounts to solving:

- LPs (Bennet and Mangasarian, 1992)
- Convex Programs (Falk and Palocsay, 1994)

LINEARLY INSEPARABLE SETS

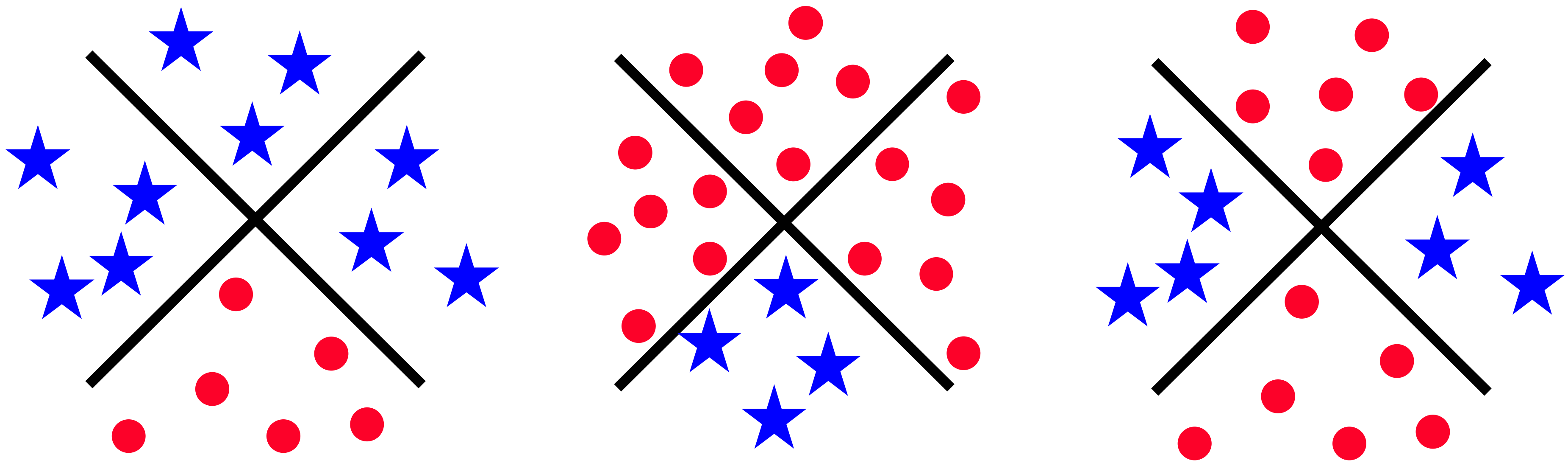


BILINEARLY SEPARABLE SETS



***NP*-hard Problem (Falk and Palocsay,1994)**

BILINEAR (IN-)SEPARABILITY OF TWO SETS IN R^n



Requires the solution of three nonconvex bilinear programs

RESULTS ON WDBC DATABASE

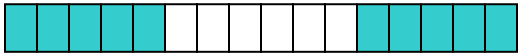
	Rows	Columns	Bilinear Terms	CPU sec
BLP1	706	350	165	11
BLP2	706	396	188	27
BLP3	1412	1432	1412	460

99% accuracy on testing set

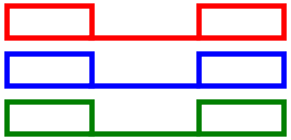
- LP-based local search to solve BLPs gives 95% accuracy

cAMP RECEPTOR PROTEIN

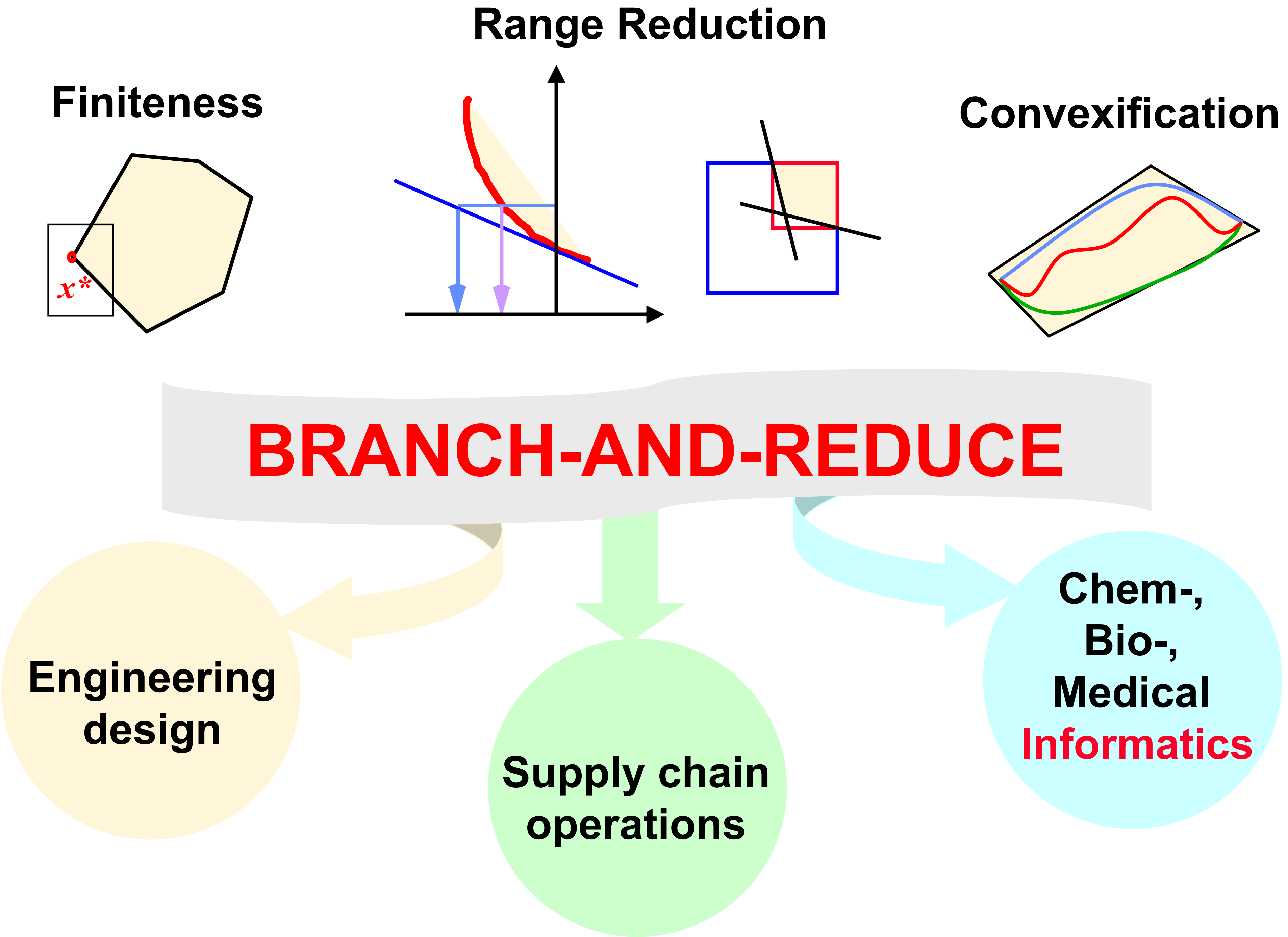
Topology of consensus pattern:



```
TAATGTTTGTGCTGGTTTTTGTGGCATCGGGCGAGAATAGCGCGTGGTGTGAAAGACTGTTTTTTTGATCGTTTTCACAAAAATGGAAGTCCACAGTCTTGACAG
GACAAAAACGCGTAACAAAAGTGTCTATAATCACGGCAGAAAAGTCCACATTGATTATTTGCACGGCGTCACACTTTGCTATGCCATAGCATTTTTATCCATAAG
ACAAATCCCAATAACTTAATTATTGGGATTTGTTATATATAACTTTATAAATTCCTAAAATTACACAAAGTTAATAACTGTGAGCATGGTCATATTTTTATCAAT
CACAAAGCGAAAGCTATGCTAAAACAGTCAGGATGCTACAGTAATACATTGATGTACTGCATGTATGCAAAGGACGTCACATTACCGTGCAGTACAGTTGATAGC
ACGGTGCTACACTTGTATGTAGCGCATCTTTCTTTACGGTCAATCAGCAAGGTGTTAAATTGATCACGTTTTAGACCATTTTTTCGTCGTGAAACTAAAAAAACC
AGTGAATTATTTGAACCAGATCGCATTACAGTGATGCAAACTTGTAAGTAGATTTCCTTAATTGTGATGTGTATCGAAGTGTGTTGCGGAGTAGATGTTAGAATA
GCGCATAAAAAACGGCTAAATTCTTGTGTAAACGATTCCACTAATTTATTCCATGTCACACTTTTCGCATCTTTGTTATGCTATGGTTATTTCATACCATAAGCC
GCTCCGGCGGGGTTTTTTGTTATCTGCAATTCAGTACAAAACGTGATCAACCCCTCAATTTTCCCTTTGCTGAAAAATTTTCCATTGTCTCCCCTGTAAAGCTGT
AACGCAATTAATGTGAGTTAGCTCACTCATTAGGCACCCCAGGCTTTACACTTTATGCTTCCGGCTCGTATGTTGTGTGGAATTGTGAGCGGATAACAATTTCAC
ACATTACCGCCAATTCTGTAACAGAGATCACACAAAGCGACGGTGGGGCGTAGGGGCAAGGAGGATGGAAAGAGGTTGCCGTATAAAGAAACTAGAGTCCGTTTA
GGAGGAGGCGGGAGGATGAGAACACGGCTTCTGTGAACTAAACCGAGGTCATGTAAGGAATTTCGTGATGTTGCTTGCAAAAATCGTGGCGATTTTATGTGCGCA
GATCAGCGTCGTTTTAGGTGAGTTGTTAATAAAGATTTGGAATTGTGACACAGTGCAAATTCAGACACATAAAAAAACGTCATCGCTTGCATTAGAAAGGTTTCT
GCTGACAAAAAAGATTAAACATACCTTATACAAGACTTTTTTTTCATATGCCTGACGGAGTTCACACTTGTAAGTTTTCAACTACGTTGTAGACTTTACATCGCC
TTTTTTAAACATTAAAATTCTTACGTAATTTATAATCTTTAAAAAAAGCATTTAATATTGCTCCCCGAACGATTGTGATTCGATTCACATTTAAACAATTTCAGA
CCCATGAGAGTGAAATTGTTGTGATGTGGTTAACCCAATTAGAATTCGGGATTGACATGTCTTACCAAAAGGTAGAACTTATACGCCATCTCATCCGATGCAAGC
CTGGCTTAACTATGCGGCATCAGAGCAGATTGTACTGAGAGTGCACCATATGCGGTGTGAAATACCGCACAGATGCGTAAGGAGAAAATACCGCATCAGGCGCTC
CTGTGACGGAAGATCACTTCGCAGAATAAATAAATCCTGGTGTCCCTGTTGATACCGGGAAGCCCTGGGCCAACTTTTGGCGAAAATGAGACGTTGATCGGCACG
GATTTTTATACTTTAACTTGTTGATATTTAAAGGTATTTAATTGTAATAACGATACTCTGGAAAGTATTGAAAGTTAATTTGTGAGTGGTCGCACATATCCTGTT
```



Some potential solutions



Range Reduction
Finiteness
Convexification
x*
BRANCH-AND-REDUCE
Engineering design
Supply chain operations
Chem-, Bio-, Medical Informatics